Methods of detection of chaotic diffusion in dynamical systems

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Introduction

- Classifying motions in regular and chaotic ones plays an important role in the study of dynamical systems.
- Nonetheless, one additional question arises: **how chaotic** the motion is?
- Processes of diffusive nature act in the the system, leading to chaotic phenomena in different timescales.
  - Nonetheless, motions deemed as chaotic might take timescales longer than the system's total lifetime to become disruptively chaotic.
  - Thus, quantifying diffusive processes and timescales is important in actually characterizing the system's phase space and different dynamical regimes of motion.

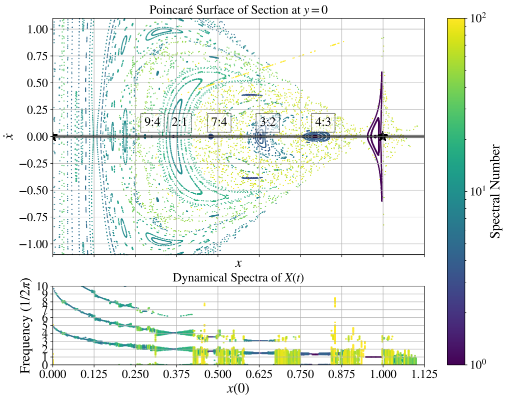
Objectives

- The purpose of this work is:
  - To characterize the Phase Space of the Planar Circular Restricted 3 Body Problem (PCR3BP),
  - To apply the several methods of chaotic motion detection, such as Poincaré Surface of Section (S.O.S.), Spectral Number (SN) and Fast Lyapunov Indicator (FLI),
  - To apply the Wavelet Analysis Method (WAM) as means of studying diffusion of Fundamental Frequencies,
  - To compare the results obtained through different tools.
- The Rotating Reference of Frame was chosen in the construction of the S.O.S. and the FLI. In the cases of the SN and WAM, coordinates in the inertial reference of frame were used.

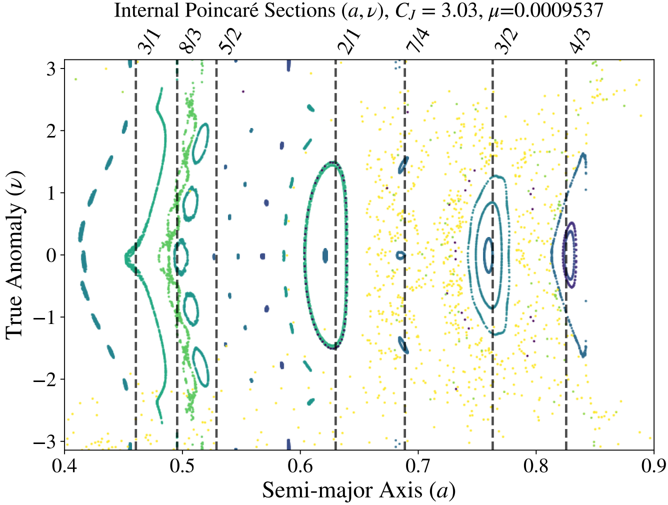
Results: Phase Space Characterization ($\mu = 0.0009537$)

Poincaré S.O.S. allows visualization of different dynamical regimes of motion:

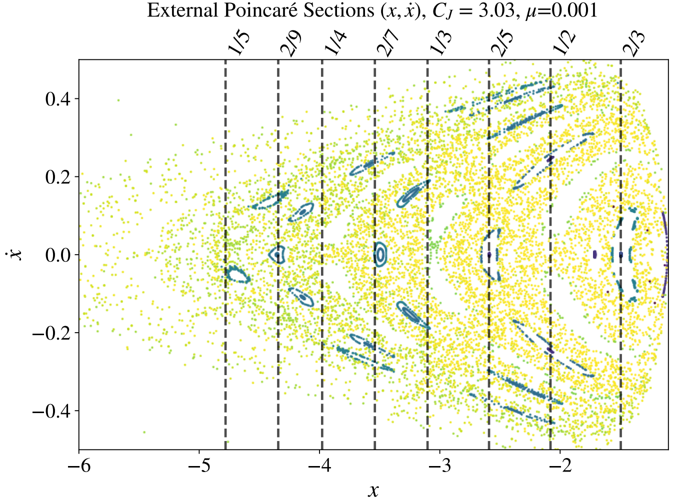
- Periodic Orbits: Fixed Points;
- Quasi-Periodic Orbits: Smooth curves (islands in the case of the resonant motion);
- Chaotic Orbits: Randomly scattered points;
- Dynamical Spectra allow us to visualize the fundamental frequencies of each region of Phase Space.



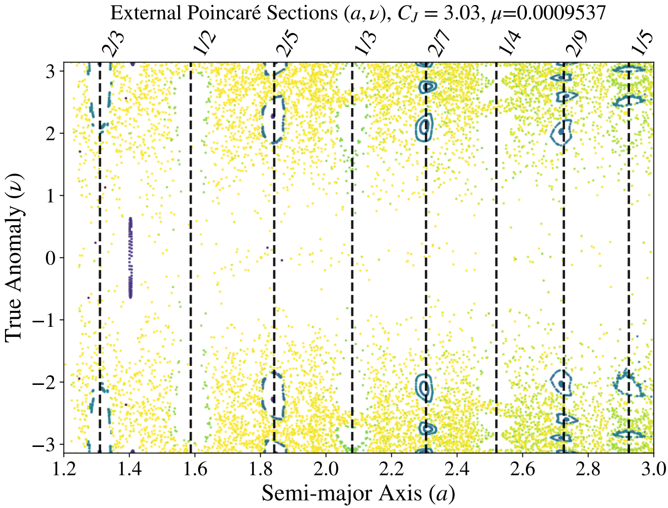
Results: Phase Space Characterization (Continuation)



Results: Phase Space Characterization (Continuation)



Results: Phase Space Characterization (Continuation)



Choice of Initial Conditions

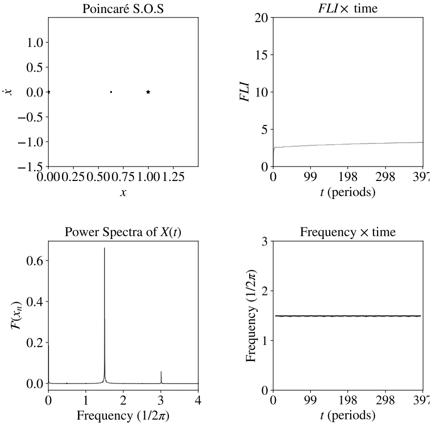
We choose four initial conditions leading to prograde orbits:

- One at the center of the $3/2$ resonance ($x(0) = 0.6277$),
- One around the $5/2$ resonance ($x(0) = 0.202$),
- One at the separatrix of the $5/2$ resonance ($x(0) = 0.2008$),
- One at the chaotic region of the Phase Space near the $3/2$ resonance ($x(0) = 0.675$),
- And one at the separatrix of $2/1$ resonance ($x(0) = 0.417$).

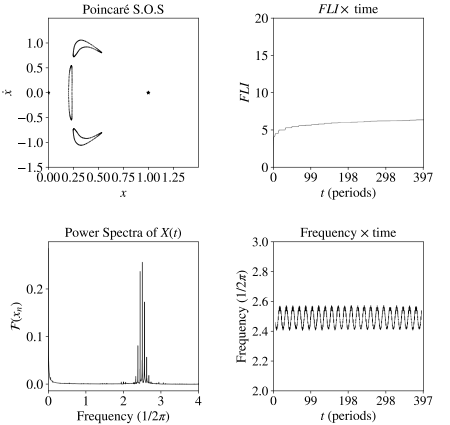
All orbits were integrated for approximately 400 periods of the binary, with $y(0) = 0, \dot{x}(0) = 0$ and $\dot{y}(0) \geq 0$ obtained for a Jacobi Constant of $C_J = 3.03$. The orbit at $x(0) = 0.417$ was integrated for ≈ 800 periods.

Results for Regular Orbits

Periodic



Quasi-periodic



Analysis for Regular Orbits

Periodic Orbit

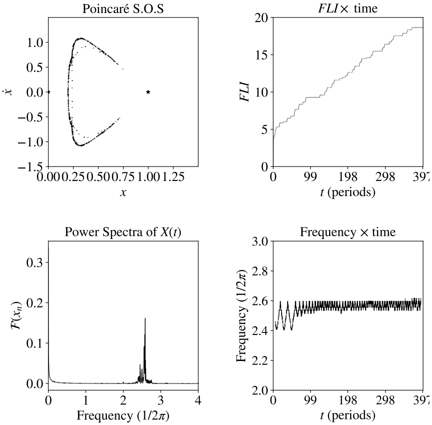
- Poincaré S.O.S. consists of singular point in the phase space,
- Discrete growth of FLI, aways under 5.
- Only one frequency and it's harmonics are detected in the power spectrum, constant in time.

Quasi-Periodic Orbit

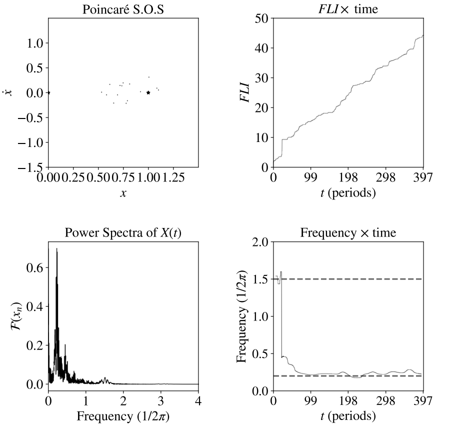
- Poincaré S.O.S. consists of three smooth resonant islands, indicating it's a resonance of order $q = 3$,
- Slightly bigger value of FLI, growing rate still small.
- Two independent frequencies and it's linear combinations,
- WAM show regular oscillation around a constant-in-time mean value (leakage from CWT).

Results for Chaotic Orbits

Periodic



Quasi-periodic



Analysis for Chaotic Orbits

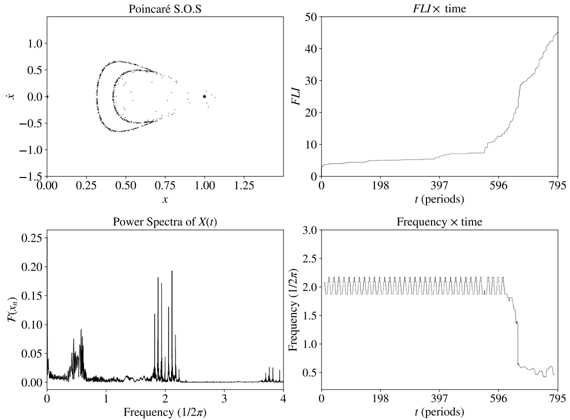
Weak Chaotic Orbit

- Points in the S.O.S scatters closely to 5/2 resonance ("sticky"),
- FLI grows continuously (but still has plenty of *plateaus*),
- Independent frequencies can't be precisely distinguished,
- WAM shows irregular, low amplitude, variations in the resonant frequency.

Strong Chaotic Orbit

- Randomly distributed points in the S.O.S., reaching outer region of phase space,
- FLI grows at greater rates, with jumps, quickly reaching $FLI = 20$
- Independent frequencies aren't discernible at all,
- WAM shows strong erratic variations, transitions and resonance captures (Particle starts near 3/2 resonance and spends some time oscillating around 1/5 resonance).

Slow Chaos



- High FLI and Spectral Number; Lyapunov indicator takes almost 600 periods to detect chaotic motion
- Points at Poincaré S.O.S. stick to 2/1 resonance,
- Almost 600 periods until extreme event happens, much higher than system's Lyapunov Time ($\approx 17 periods$)

Conclusions

- Different chaos detection tools constitutes a powerful and reliable way of studying dynamical systems.
- More specifically, using the Wavelet Analysis Method as means of studying diffusion of fundamental frequencies, system's dynamics could be studied with temporal resolution:
  - Correlations can be seen between the FLI and WAM, but no more information can be obtained from the variational method about the *plateus*.
- We are able to cast light to the time evolution of those systems, highlighting complex phenomena in the timespan of the study and possible mechanisms leading to chaotic behaviour of the system, such as resonance transitions and captures.

References

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