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EQUILIBRIUM POINTS AROUND A NON-AXIALLY SYMMETRIC BODY

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INTRODUCTION

It is know that the circular restricted three-body problem is not integrable. However there are some special analysis that can be done to extract some information about this system. The Equilibrium Points are probable the most notable tool for such purpose. They are defined as points in the rotational frame where a particle can be placed in rest and it will remain stationary, i. e., with zero velocity and acceleration.

In this work, we propose to explore the stability around a non-axially symmetric body through a variation of the Lagrangian (or equilibrium) Points. The motivation is to study non-axially symmetric bodies and its surroundings. Haumea, for instance, is so elongated that the points L4 and L5 are not stable. Also, its rings, as well as Chariklo's rings, are far from the corotation radius and near to the Roche limit (Sicardy 2018).

METODOLOGY

Spherical body of radius R and mass M

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Topographic anomaly of mass μ (relative to the spherical body) and located at the equator.

The potential of a spherical body with a topographic feature considering the radiation solar pressure acting on a particle is:

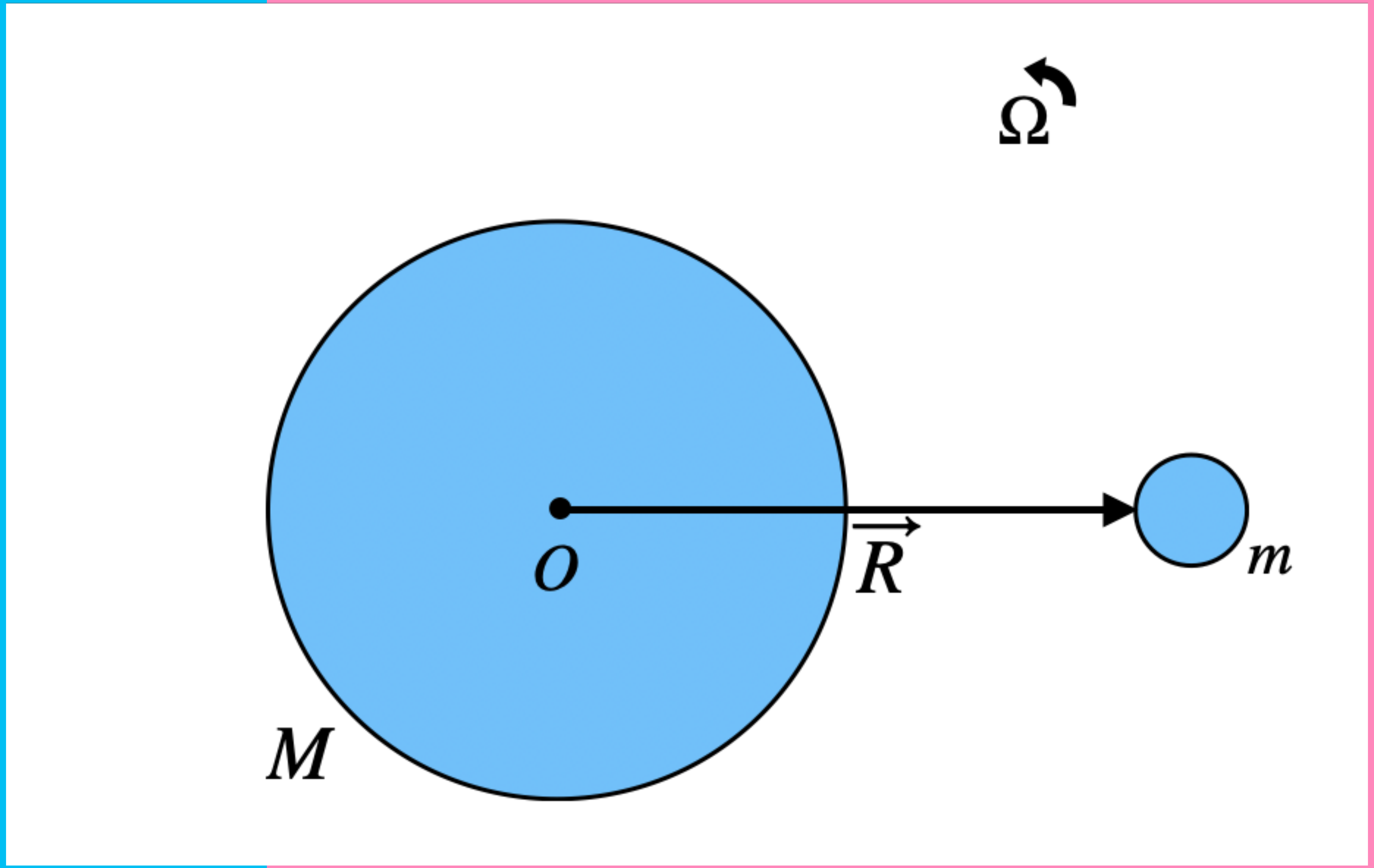
$$V(\vec{r}) = -\frac{GM(1-\beta)}{r} - GM\mu\left(\frac{1}{\Delta} - \frac{qRr\cos\theta}{R^3}\right) - \frac{\Omega^2 r^2}{2}.$$

The parameter q is the ratio between the centrifugal and the gravitational forces, for a body to be stable it is required q < 1.

$$q = \frac{\Omega^2 R^3}{GM}$$

$$\beta = \frac{F_{rad}}{F_{gr}} = \frac{(A/m)Q_{pr}L_0}{4\pi cm}$$

β is the ratio of two forces, radiation and gravitational, where A is the geometric cross–section of a particle with mass m, c is the speed of light, Q_{pr} is the radiation pressure coefficient and L_0 is the luminosity of the Sun.



- **O the origin of the coordinate system that matches with the spherical body center;**
- **$\vec{R}$ is the position vector of the mass anomaly from the center O;**
- **The whole body rotates with an uniform rate Ω;**

ANALYTICAL APPROACH

From the potential we can derive the equilibrium locations by isolating the azimuthal component and calculating the critical points.

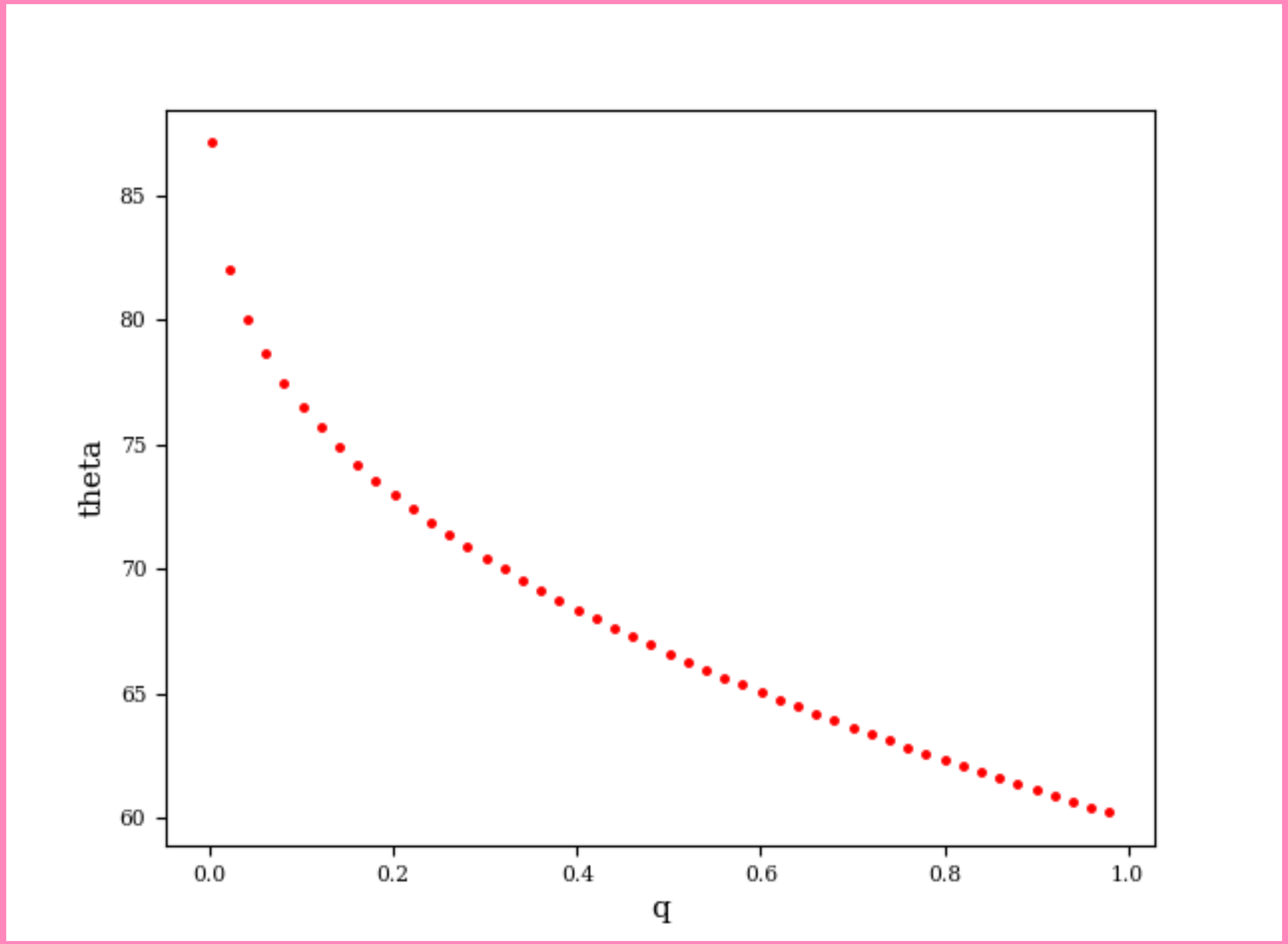
$$V(\vec{r}) = -\frac{GM(1-\beta)}{r} - GM\mu\left(\frac{1}{\Delta} - \frac{qRr\cos\theta}{R^3}\right) - \frac{\Omega^2 r^2}{2}.$$

$$f(\theta) = \frac{1}{\Delta} - \frac{qRr\cos\theta}{R^3},$$

$$\boxed{\cos\theta = \frac{q^{-1/3}(1-\beta)^{1/3}}{2}.}$$

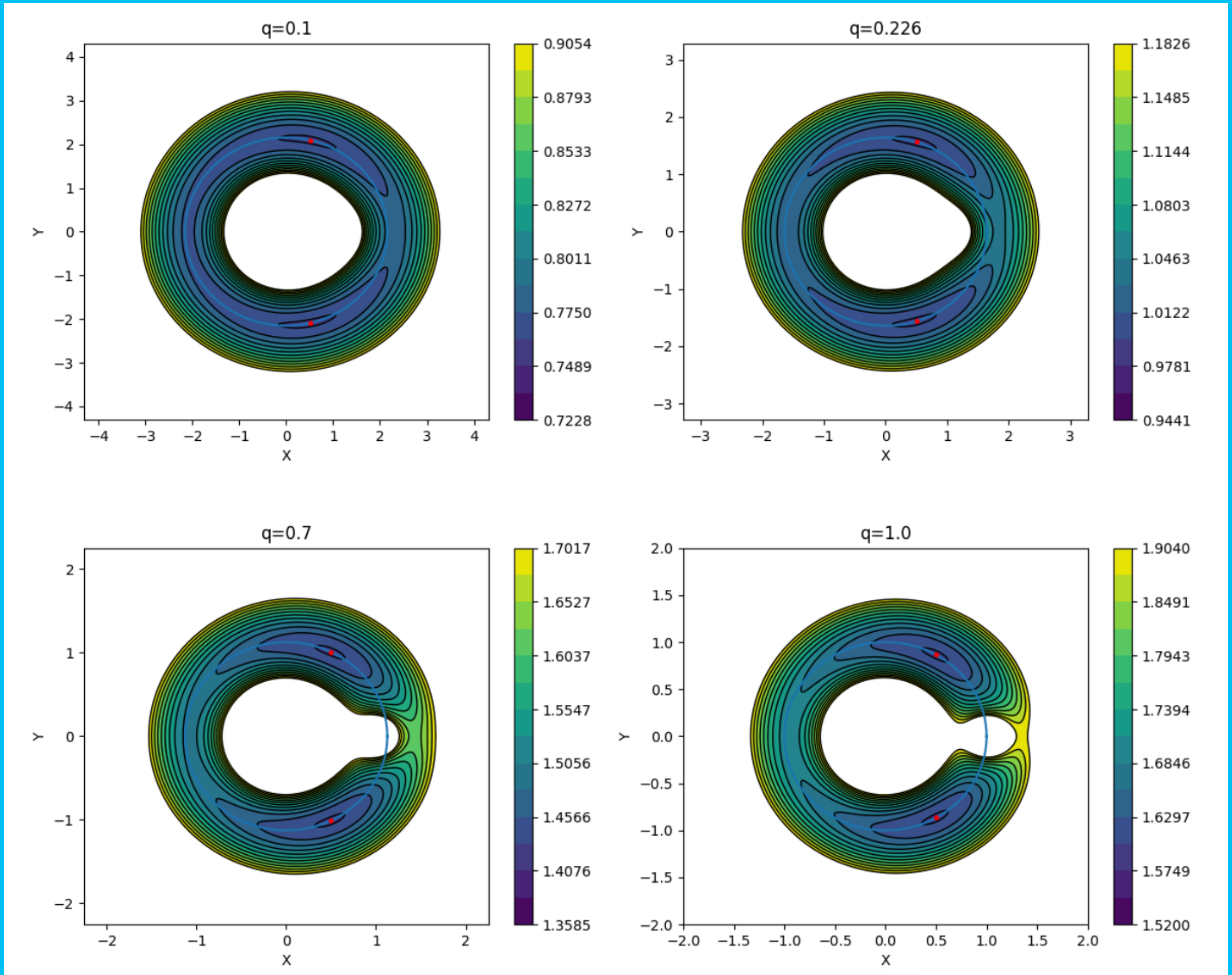
$$\theta = \arccos\left(\frac{q^{-1/3}(1-\beta)^{1/3}}{2}\right), \quad \text{for} \quad q = 1 : \begin{cases} \boxed{\text{if} \quad \beta \to 0, \quad \theta \to \pi/3 \qquad \text{Classical case}} \\ \text{if} \quad \beta \to 1, \quad \theta \to \pi/2. \end{cases}$$

$$\cos\theta = \frac{q^{-1/3}(1-\beta)^{1/3}}{2}$$



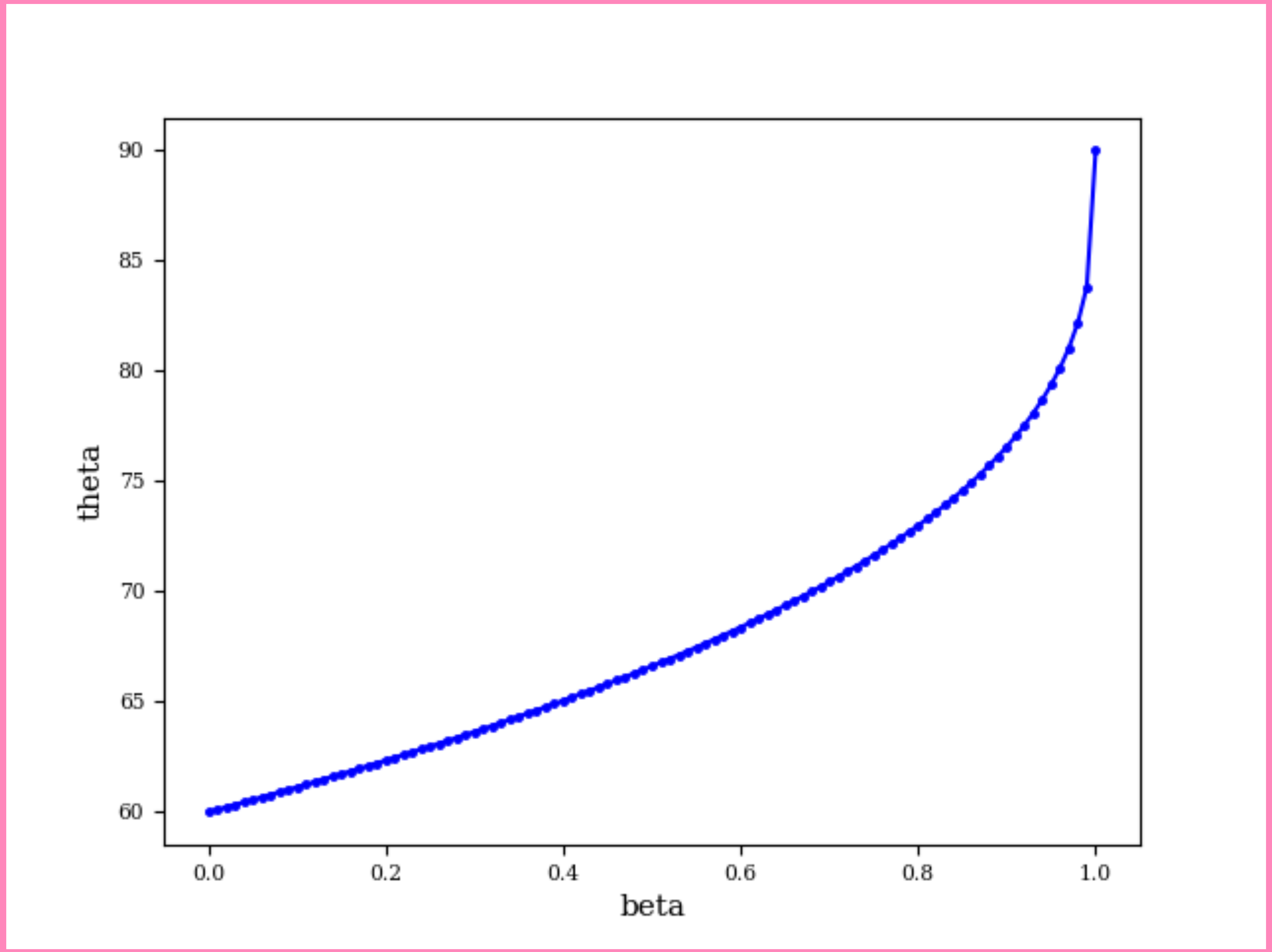
When the solar radiation is neglected, and for values of q from 1 to 0 the tendency of θ is go from 60 to 90 degrees.

The figure presents four sets of ZVC for different values of q. The values of the potential are shown in the vertical color bar. The blue circles indicate the corotation distance and the plots axes are adapted to keep it fixed on the frames. The red dots marks the equilibrium points, C4 and C5 (alternates for L4 and L5).



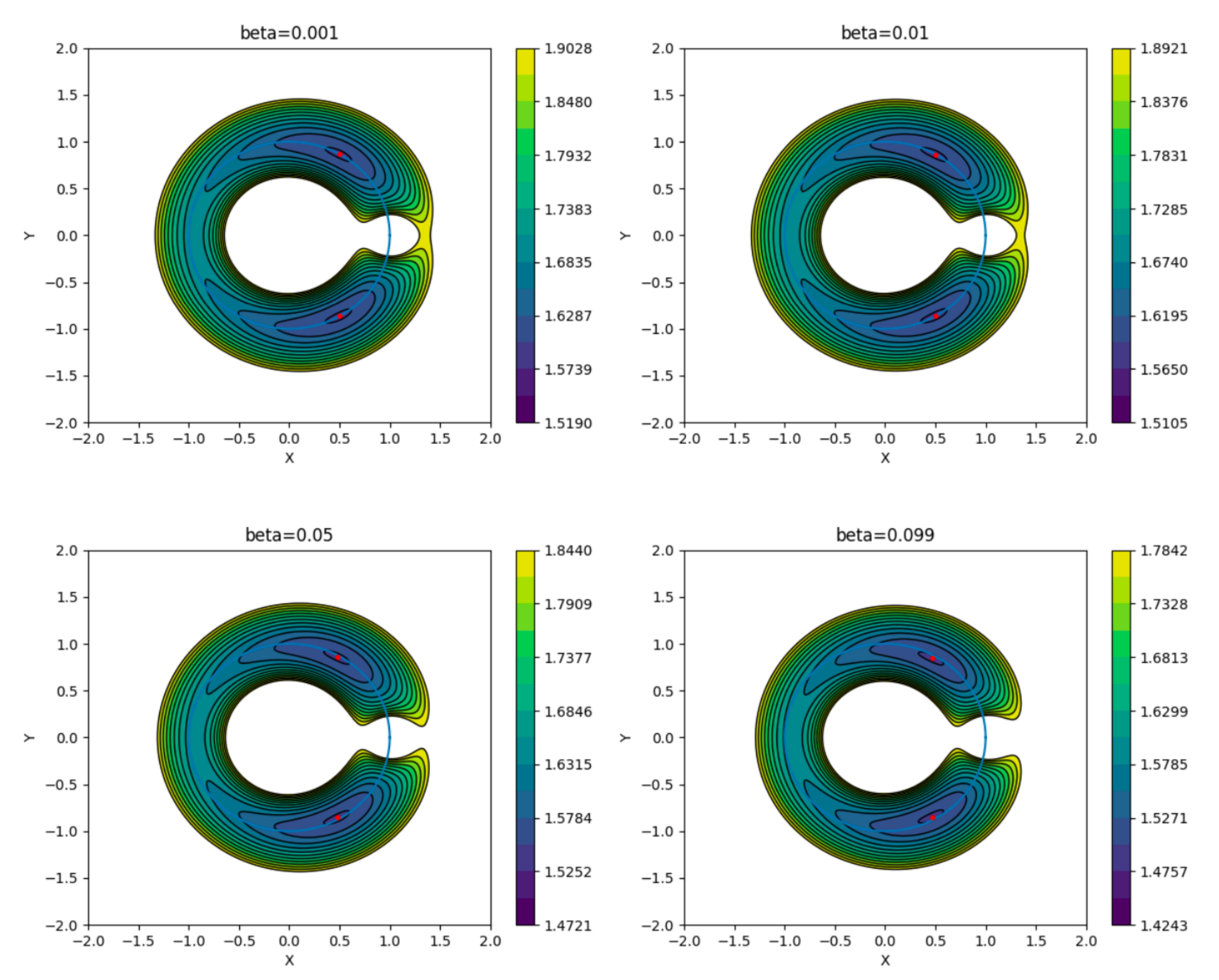
ZERO VELOCITY CURVES

$$\cos\theta = \frac{q^{-1/3}(1-\beta)^{1/3}}{2}$$



For the classical case of $q = 1$, and for values of β from 0 to 1 the tendency of θ is also go from 60 to 90 degrees.

The figure presents four sets of ZVC for different values of β. The values of the potential are shown in the vertical color bar. The blue circles indicate the corotation distance and the plots axes are adapted to keep it fixed on the frames. The red dots marks the equilibrium points, C4 and C5 (alternates for L4 and L5).



ZERO VELOCITY CURVES

FINAL REMARKS AND NEXT STEPS

- We derived an analytical approach to locate the equilibrium points around a body with a topography feature;
- And also investigated the changes of this location due the radiation pressure;
- A numerical routine was implemented to compute the ZVC of the system;
- A iterative correction for a second order on the values of theta and r are under development;
- Additionally, it may be interesting to explore such configurations once they are not into the Half-Gashau limit of mass;

THANK YOU!

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