On the applicability of Benford law to exoplanetary and asteroid data

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Preliminary

Benford's Law was formulated by the Physicist Frank Benford in 1938 [2], although was originally proposed by the Astronomer Simon Newcomb in 1881 [1]. It states that the frequency of a distribution of the first significant digit in a big data set follows the following law

$$P^{B}_{\eta_1} = \log_{10}(1+\eta_1) - \log_{10}(\eta_1) \tag{1}$$

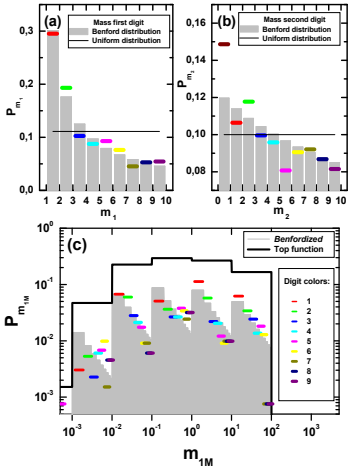
where η_1 is the first non-null digit of a given value η and $P^{B}_{\eta_1}$ is the Benford probability of occurring such a digit.

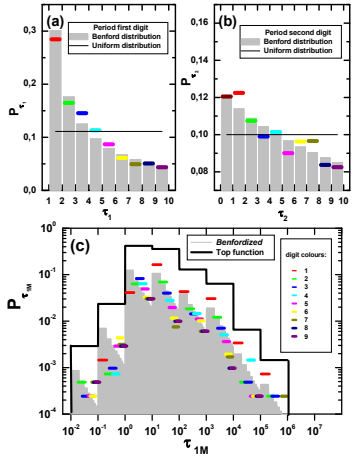
Benford Law will not work for dataset with small sizes.

The level of the uncertainty of the data presents an obvious limitation for the applicability of Benford' Law since only digits beyond the error level should be considered.

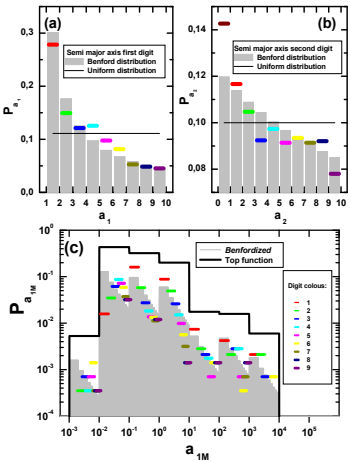
Details in: **Melita M.D. & J.Miraglia, New Astronomy, Volume 89, article id. 101654.**

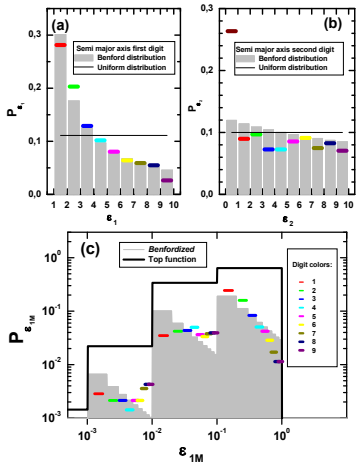
Exoplanet's Masses and Periods



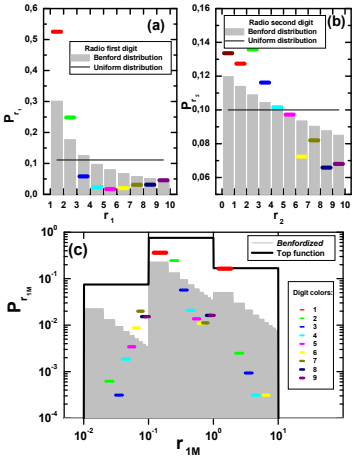


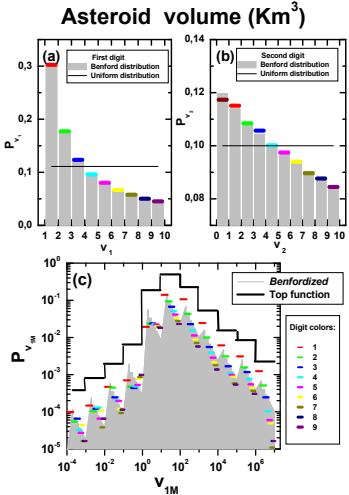
Exoplanet's Semimajor axes and eccentricities





Exoplanet's Radii and Asteroid's Volumes





Discussion

Benford's Law has proven to be a successful tool to diagnose data sets. In the case of natural sciences a departure from the predicted distribution can imply a lack of precision or model-dependent biases.

In the case of the fundamental parameters of the exoplanets data set, we find that the radios are affected by the small rank, due to obvious observational selection effects, and the eccentricities exhibit a lack of precision. The rest of the parameters show a reasonable compliance with BL.

The analysis order-by-order of magnitude has shown that eccentricities, semimajor axes, masses and periods tend to show a lack of completeness in the smallest orders of magnitude. As mentioned, the case of the radios a bias related with modeling might be operating in the data set. When we discriminate by discovery method, we find that Primary Transit produces less biases in all parameters except in the case of the semi major axes where the fit with BL is better in the case of the Radial Velocity method, which may imply that, for this parameter, modeling has a smaller influence. We also find that the volumes -but not the radii- of the asteroids satisfy very well the BL.

Conclusion

We note a plot of exoplanetary radii as a functions of masses starts to saturate around 0.2 Jupiter masses and it becomes quite flat for larger masses. In practice, objects between $0.2 - 100$ Jupiter masses have more or less the same physical radii, with values between $0.8 - 2$. Jupiter-radii. Therefore, it is expected expect that Benford's law would fail naturally. To check if the disagreement occurs evenly in all the range of sizes, we divided the sample at a value of 0.7 Jupiter radius, and we associate objects with the larger sizes with Gas-Giants and those with the smaller sizes with Super-Earths. We find that the disagreement in the distribution of the first digit for the radius of Super-Earths is considerably smaller than that of the Gas-Giants, where the rms deviation of the former is ~ 0.05 and the one of the later is ~ 0.18. Therefore, the disagreement found for exoplanetary radii can be attributed to the saturation of values beyond 0.2 Jupiter masses.

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