“Polarimetry of Solar System objects”

1: Introduction to astronomical polarimetry

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Why polarimetry?

• Wherever there is appreciable asymmetry in an astronomical situation, there is likely to be polarization at some level.
• The character of the asymmetry determines the kind of polarization to be expected:

- If the asymmetry is of scalar kind, the polarization produced will be circular.
- If the asymmetry is of vectorial kind, the polarization produced will be linear.

•The main asymmetries giving rise to astronomical polarization are magnetic fields and an asymmetric distribution of scattered radiation.

Why polarimetry?

• In the case of scattered radiation, the direction of vibration of the electric vector is at right angles to the scattering plane, the plane containing the incident and the scattered rays.

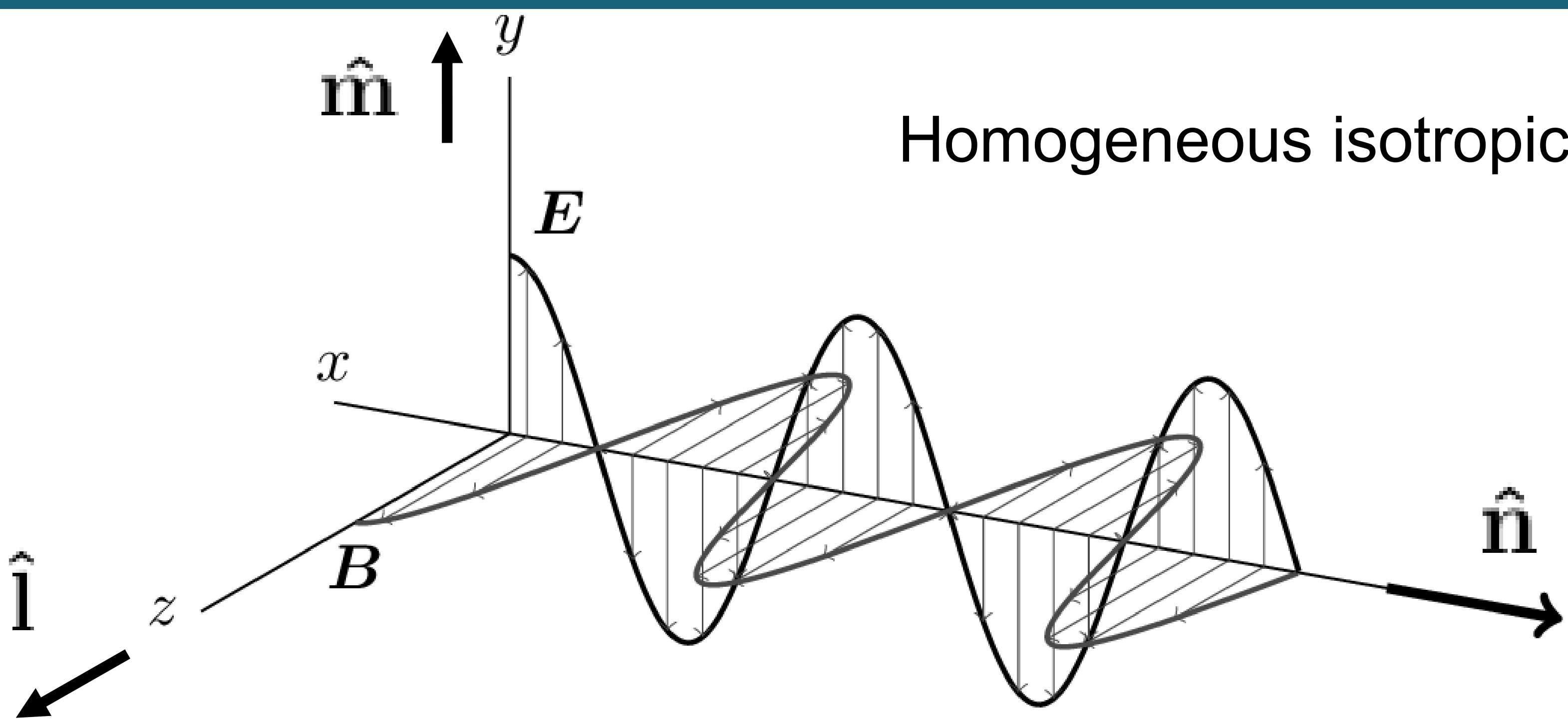
• Measurements of the linear polarization can help to identify the scattering mechanism or...

• they can give information on the properties of the source (e.g. orientation as projected on the sky, spottedness) and/or the scattering medium (e.g. size, shape, degree of alignment and refractive index of the particles).

Literature

- “Polarization of Starlight” by K. Serkowski. Adv. In Astron. Astrophys. 1, 289, 1962.
- “Planets, Stars and Nebulae Studied with Photopolarimetry” by T. Gehrels, University of Arizona Press, 1974.
- “Astronomical polarimetry” by Jaap Tinbergen. Cambridge University Press, 1996.
- “Polarimetric remote sensing of solar system bodies” by Mikhaylo Mishchenko et al. . Akademperiodyka, 2010.
- “Stellar polarimetry” by David Clarke. Willey-VCH Verlag GmbH & Co., 2010.
- “Polarimetry of Stars and Planetary Systems“ by L. Kolokolova, J. Hough and A-. C. Levasseur-Regourd. Cambridge University Press, 2015.

Waves



Homogeneous isotropic medium

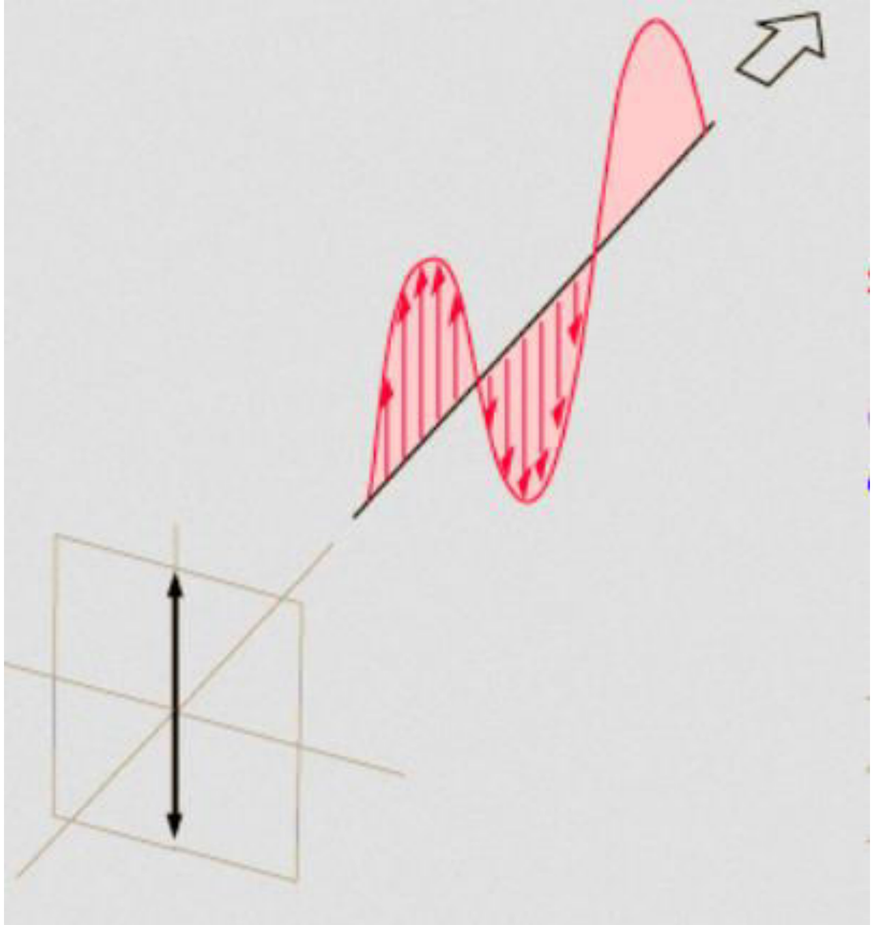
$$c = \frac{E}{B}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

λ = constant ⟶ monochromatic wave

Plane waves

The wave displacement is always in the same plane.



$$\vec{\mathbf{E}} = A\hat{\mathbf{a}}\exp[i(kx - \omega t + \phi)]$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{k\lambda}{T} = kc$$

Plane waves are polarized waves

Polarization

$$\vec{\mathbf{E}} = E_1\,\hat{\mathbf{l}} + E_2\,\hat{\mathbf{m}}$$

$$E_1 = A_z \exp(i\phi_z) \exp[i(kx - \omega t)]$$

$$E_2 = A_y \exp(i\phi_y) \exp[i(kx - \omega t)]$$

$$x = 0 \quad \rightarrow$$

$$\vec{\mathbf{E}} = A_z\,\hat{\mathbf{l}} \exp[i(\phi_z - \omega t)] + A_y\,\hat{\mathbf{m}} \exp[i(\phi_y - \omega t)]$$

$$E'_z = A_z \exp[i(\phi_z - \omega t)] \;\rightarrow E_z = Re[E'_z] = A_z \cos(\phi_z - \omega t)$$

$$E'_y = A_y \exp[i(\phi_y - \omega t)] \;\rightarrow E_y = Re[E'_y] = A_y \cos(\phi_y - \omega t)$$

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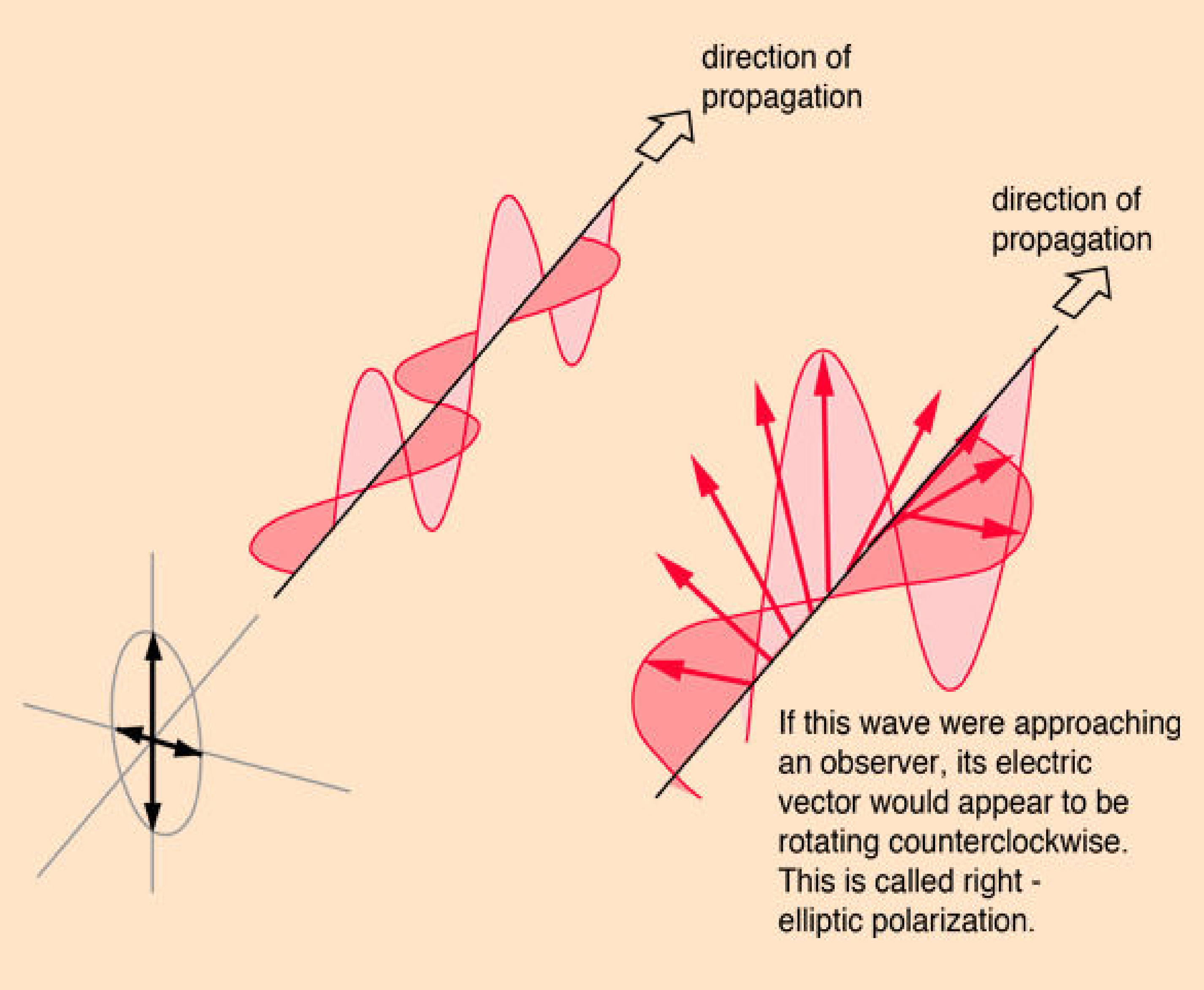
equation of an ellipse in polar coordinates

Elliptic polarization



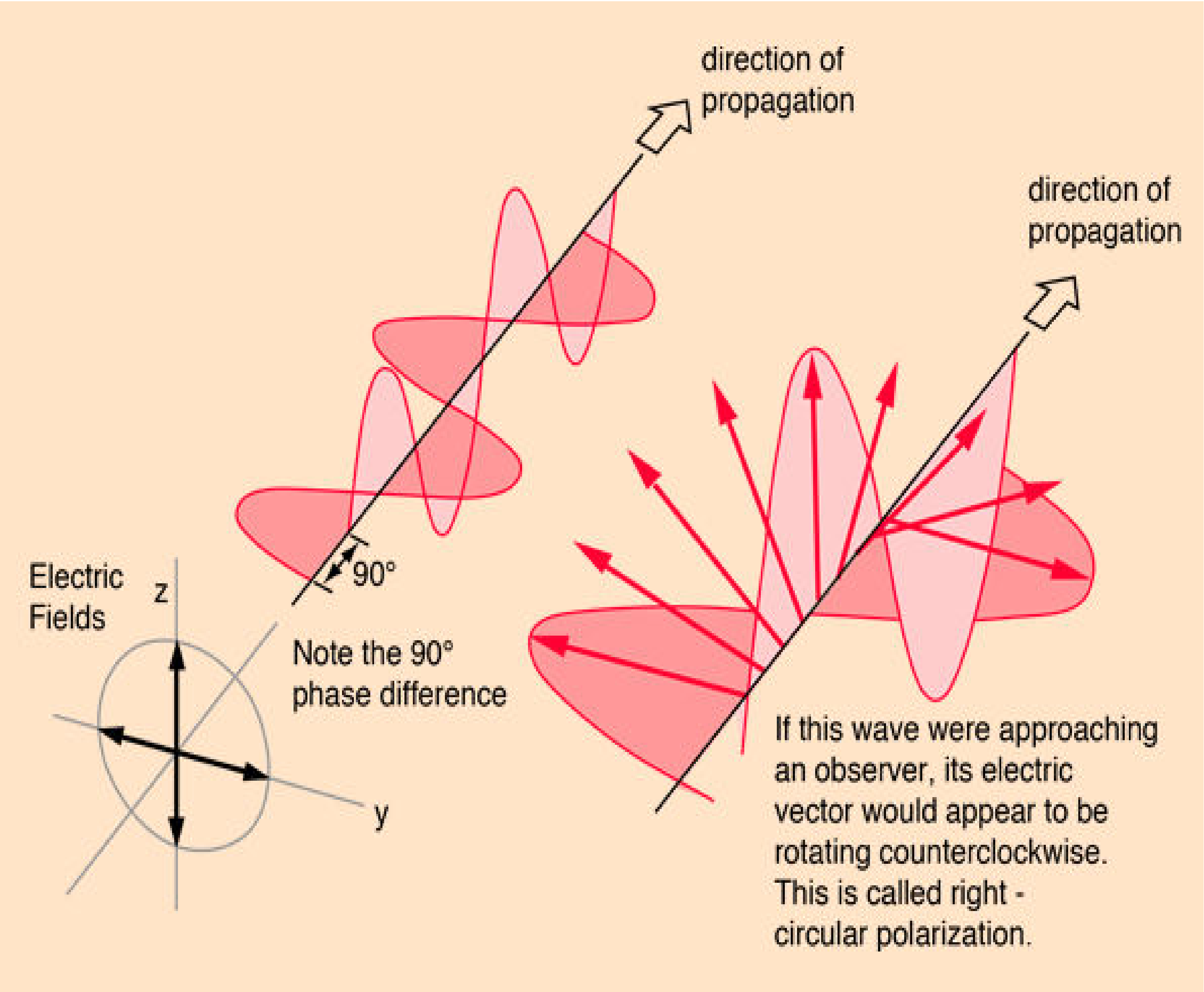
$$E_z = A_z \cos(\phi_z - \omega t)$$
$$E_y = A_y \cos(\phi_y - \omega t)$$

$$\phi_y - \phi_z \neq 0$$
$$\phi_y - \phi_z \neq \pm\frac{\pi}{2}$$
$$A_y \neq A_z$$

Two tipes:

right and left elliptic polarization

Circular polarization



$$E_z = A_z \cos(\phi_z - \omega t)$$
$$E_y = A_y \cos(\phi_y - \omega t)$$

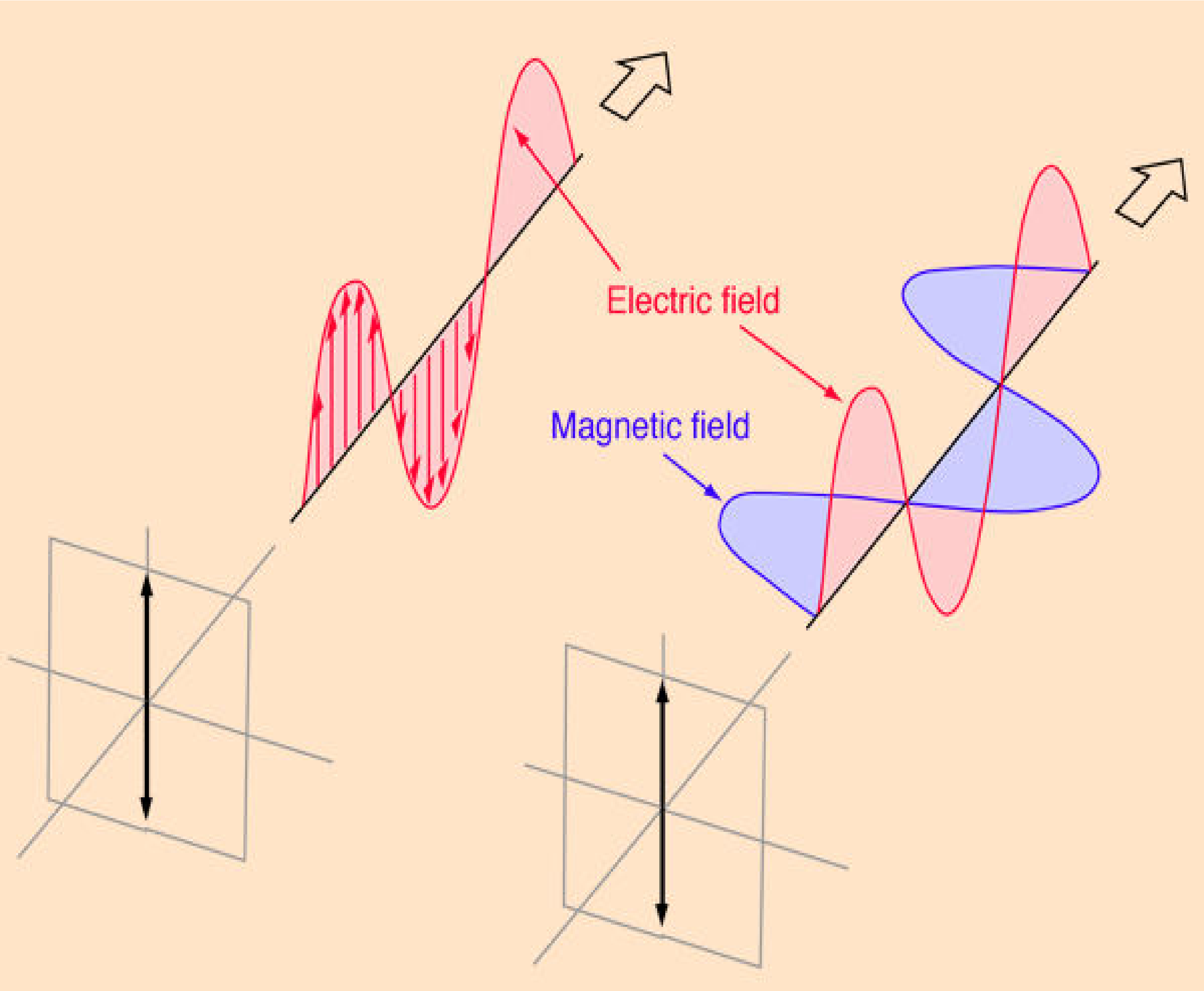
$$\phi_y - \phi_z = \pm \frac{\pi}{2}$$

$$A_y = A_z$$

Two tipes:

right and left circular polarization

Linear polarization



$$E_z = A_z \cos(\phi_z - \omega t)$$

$$E_y = A_y \cos(\phi_y - \omega t)$$

$$\phi_y - \phi_z = 0$$

Most usual tipe of polarization

Monochromatic polarization modes

- Linear:

Electric field with constant orientation; strength strictly sinusoidal with time; Constant amplitude and frequency.

- Circular:

Can be seen as a combination of two linearly polarized waves with vibration directions at right angles of each other with equal amplitude and differing 90^o in phase. Electric vector of constant magnitude but its orientation moves uniformly with time making one revolution per wave period. Two tipes: left and right.

- Elliptical:

The most general form. Can be seen as a combination of two linearly polarized waves with vibration directions at right angles of each other with a phase difference something other than 0^o or 90^o. There are several possibilities. Two tipes: left and right.

Stokes parameters

- One of several possible representations.
- Proposed by George Stokes in 1852.
- Introduced in astronomy by Subrahmanyan Chandrasekhar in 1946.
- They are often gathered into a four elements vector labelled I, Q, U, V.
- Other possible notations: (S_0, S_1, S_2, S_3) or (I, M, C, S).

$$
\begin{aligned}
I &= < E_2 E_2^* + E_1 E_1^* > &= &< A_y^2 + A_z^2 > \\
Q &= < E_2 E_2^* - E_1 E_1^* > &= &< A_y^2 - A_z^2 > \\
U &= < E_2 E_1^* + E_1 E_2^* > &= &< 2 A_y A_z \cos(\phi_z - \phi_y) > \\
V &= i < E_2 E_1^* - E_1 E_2^* > &= &< 2 A_y A_z \sin(\phi_z - \phi_y) >
\end{aligned}
$$

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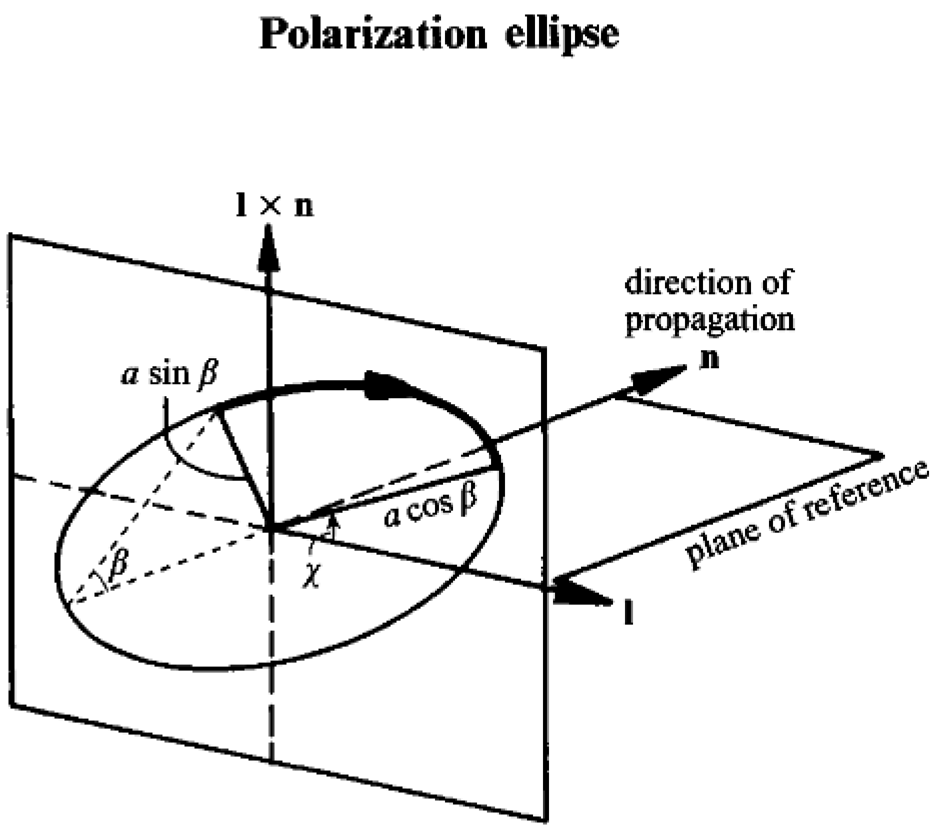
$$
\begin{aligned}
I &= I_0 + I_{90} \\
Q &= I_0 - I_{90} \\
U &= I_{45} - I_{135} \\
V &= I_{rc} - I_{lc}
\end{aligned}
$$

Stokes parameters



$$
\begin{aligned}
I &= a^2 \\
Q &= < a^2 \cos 2\beta \cos 2\chi > \\
U &= < a^2 \cos 2\beta \sin 2\chi > \\
V &= < a^2 \sin 2\beta >
\end{aligned}
$$

χ = polarization angle

tan β = axial ratio

I >= 0

Q, U, V can be positive or negative

Stokes parameters

Linear polarization:

- $\sin \beta = 0$
- $\sin 2\beta = 0$, so $V=0$
- $Q = a^2 \cos 2\chi$
- $U = a^2 \sin 2\chi$
- a^2 is the intensity
- χ is the orientation of the ellipse (in this case, a line).
- χ is called azimuth or polarization angle.
- Q and U are cartesian components of the vector (a^2, 2χ).
- a Q vs. U diagram is a common representation in astronomy.
- the origin of χ is chosen arbitrarily, but in a Q vs. U diagram the origin of 2χ is by definition de Q-axis.

Stokes parameters

Circular polarization:

- sin β = +- cos β
- sin 2β = +- 1 , |V| = I
- Q = 0, U = 0
- the sign convention is arbitrary. In fact, radio and optical polarimetry use contrary conventions.

Stokes parameters

Elliptical polarization:

- tan β is the axial ratio of the ellipse.
- Q, U, and V are non-zero values.
- $Q^2 + U^2 + V^2 = I^2$
- the sign convention is arbitrary. In fact, radio and optical polarimetry use contrary conventions.

Stokes parameters

Table 2.1 *Representative Stokes vectors, after Shurcliff (1962)*

Polarization form		Ellipse parameters		Amplitude/phase		Normalized Stokes vector
Pattern	L/C/E/U l/r	χ (deg)	$\tan\beta$	A_y/A_x	$\phi_y - \phi_x$ (deg)	I, Q, U, V
—	L	0	0	0	–	1, 1, 0, 0
|	L	90	0	∞	–	1, −1, 0, 0
/	L	45	0	1	0	1, 0, 1, 0
\	L	−45	0	1	±180	1, 0, −1, 0
/	L	any	0	>0	0 or ±180	$1, \cos 2\chi, \sin 2\chi, 0$
○	C,r	–	1	1	90	1, 0, 0, 1
○	C,l	–	1	1	−90	1, 0, 0, −1
⬭	E,r	0	0.5	0.5	90	1, 0.6, 0, 0.8
⬯	E,l	0	−2	2	−90	1, −0.6, 0, −0.8
⬯	E,r	90	2	2	90	1, −0.6, 0, 0.8
⬭	E,r	45	$\tan\beta$	1	2β	$1, 0, \cos 2\beta, \sin 2\beta$
⬭	E,r	22.5	0.318	0.518	45	$1, \sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3}$
–	U	–	–	–	–	1, 0, 0, 0

Birefringence

• For every polarization mode it is possible to define one with the same I but opposite Q, U, and V, adding or substracting 90° to β.

• In this case, the axial ratio is the same with the axis at right angles and the ellipse is traced in the opposite direction.

• Such pair of opposite modes are called orthogonal to each other. The orthogonal modes that can propagate through a medium without changing their polarimetric mode are called eigenmodes.

• If the orthogonal modes travel in a medium at different velocities (it has two refractive indices), such a medium is said to be birefringent.

• There are several birefringent cristals which are used in optical polarimetry instruments.

Unpolarized radiation

- If we take the average of all the polarization modes produced during a sufficiently “long” time interval, all values of β and χ will occur.

- This situation results in a complete combination of orthogonal polarization modes.

- In this case, the time-averaged Stokes parameters are $(\langle a^2 \rangle, 0, 0, 0)$.

- Since no single polarization mode dominates or is absent, we call such radiation as unpolarized.

- If a measurement yields such result, no matter what polarization mode one tries to detect, the radiation under scrutiny is said to be unpolarized.

Partial polarization

- When we combine waves of orthogonal polarization modes with no long-term persistence in the phase relation between them a incoherent sum is produced and Stokes vector addition would not apply.

- Partially polarizated radiation is the incoherent sum of an unpolarized and a fully polarized component.

- The Stokes parameters in this case are the sums of the Stokes parameters of the components: the I values add (they are always positive), while the Q, U, and V values are those of the fully polarized radiation.

Partial polarization

- Therefore, for partially polarized radiation, $Q^2 + U^2 + V^2 < I^2$.
- The degree of polarization is $p = (Q^2 + U^2 + V^2)^{1/2} / I$.
- The degree of linear polarization is $p_{lin} = (Q^2 + U^2)^{1/2} / I$.
- The degree of circular polarization is $p_{cir} = V / I$.
- It is usual to say that partially polarized and unpolarized radiation are in a mixed state of polarization, while fully polarized radiation is said to be in a pure state.